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DARK MATTER AND THE SCALE OF SUSY BREAKING

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We consider the prospects for dark matter (DM) in two classes of supersymmetric (SUSY) models which are characterized by the different mechanism of SUSY breaking, namely the more common supergravity models with very large scale of SUSY breaking and the recent schemes where SUSY is broken at a relatively low scale and gravitinos are likely to be the lightest SUSY particle. We point out that the former scheme is in general associated with the cold dark matter (CDM) scenario, while the latter predicts a warm dark matter (WDM) dominated Universe.

1 Introduction

We have strong indications that ordinary matter (baryons) is insufficient to provide the large amount of non-shining matter (dark matter DM) which has been observationally proved to exist in galactic halos and at the level of clusters of galaxies [1]. In a sense, this might constitute the "largest" indication of new physics beyond the standard model (SM) for electroweak interactions. This statement holds true even after the recent stunning developments in the search for non-shining baryonic objects. In September 1993 the discovery of massive dark objects ("machos") was announced. After three years of intensive analysis it is now clear that in any case machos cannot account for the whole dark matter of the galactic halos [2].

It was widely expected that some amount of non–shining baryonic matter could exist given that the contribution of luminous baryons to the energy density of the Universe $\Omega_0 = \rho/\rho_{cr}$ ($\rho_{cr} = 3H_0^2/8\pi G$ where G is the gravitational constant and H_0 the Hubble constant) is less than 1%, while from nucleosynthesis we infer $\Omega_{baryon} = \rho_{baryon}/\rho_{cr} = (0.06 \pm 0.02)h_{50}^{-2}$, where $h_{50} = H_0/(50Kms^{-1}Mpc^{-1})$. On the other hand, we have direct indications[3] that Ω_0 should be at least 20% which means that baryons can represent no more than half of the entire energy density of the Universe [1].

We could make these considerations about the insufficiency of the SM to obtain a large enough Ω_0 more dramatic if we accept the theoretical input that the Universe underwent some inflationary era which produced Ω_0 extremely close to unity. In that case, at least 90% of the whole energy density of the Universe should be provided by some new physics beyond the SM.

Before discussing possible particle physics candidates, it should be kept in mind that DM is not only called for to provide a major contribution to Ω_0, but also it has to provide a suitable gravitational driving force for the primordial density fluctuations to evolve into the large-scale structures (galaxies, clusters and superclusters of galaxies) that we observe today [1]. Here we encounter the major difficulties when dealing with the two "traditional" sources of DM: cold (CDM) and hot (HDM) DM.

Light neutrinos in the eV range are the most typical example of HDM, being their decoupling temperature of O(1 MeV). On the other hand, the lightest supersymmetric particle (LSP) in the tens of GeV range is a typical CDM candidate. Taking the LSP to be the lightest neutralino, one obtains that when it decouples it is already non-relativistic, being its decoupling temperature typically one order of magnitude below its mass.

Both HDM and CDM have some difficulty to correctly reproduce observations related to the distribution of cosmic structures (galaxies and galaxy clusters) at different scales and at different redshifts. The conflict is more violent in the case of pure HDM. Neutrinos of few eV's erase by free–streaming density fluctuations on small (galactic) scales, thus producing a wrong galaxy clustering pattern, as well as a too late galaxy formation. The opposite problem arises with pure CDM: we obtain too much power in the spectrum at small mass scales.

A general feature is that some amount of CDM should be present in any case. A possibility which has been envisaged is that after all the whole Ω_0 could be much smaller than one, say 20% or so and then entirely due to CDM. However, if one keeps on demanding the presence of an inflationary epoch, then, according to the standard picture, it seems unnatural to have Ω_0 so different from unity.

Another possibility is that CDM provides its 20% to Ω_0, while flatness is provided by a non vanishing cosmological constant. Needless to say, not having even a satisfactory reason for a vanishing cosmological constant, it seems harder to naturally obtain some specific non-vanishing value for it.

Finally, a further possibility is given by the so-called mixed dark matter (MDM) scenario[4], where a wise cocktail of HDM and CDM is present. An obvious realization of a MDM scheme is a variant of the minimal supersymmetric standard model (MSSM)[5] where neutrinos get a mass of few eV's. In that case the lightest neutralino (which is taken to be the LSP) plays the role of CDM and the light neutrino(s) that of HDM. With an appropriate choice of the parameters it is possible to obtain contributions to Ω_0 from the CDM and HDM in the desired range.

In the first part of this talk we will briefly review the "standard" situation which occurs in supergravity models where local SUSY is broken at a very high scale and gravity constitutes the messenger of SUSY breaking to the observable sector. In this frame the LSP is likely to be the lightest neutralino with typical mass in the tens of GeV. In the second part we will move to newer schemes where the breaking of SUSY occurs at scale not far from the electroweak scale and gauge forces instead of gravity are invoked as messengers of the SUSY breaking to the observable sector. This frame of SUSY breaking entails major differences on the DM problem. Indeed, gravitinos turn out to be quite light in this context and they are likely to constitute the new LSP. We will discuss the implications of these gauge-mediated SUSY breaking models for DM schemes.

2 High-Scale Susy Breaking and DM

In N=1 supergravity models [6], where a discrete symmetry, matter R–parity, discriminates between ordinary and SUSY particles, the lightest SUSY particle (LSP) is absolutely stable. For several reasons the lightest neutralino is the favourite candidate to be the LSP fulfilling the role of CDM [7].

The neutralinos are the eigenvectors of the mass matrix of the four neutral fermions partners of the W_3, B, H_1^0 and H_2^0. There are four parameters entering this matrix: M_1, M_2, μ and $tg\beta$. The first two parameters denote the coefficients of the SUSY breaking mass terms $\tilde{B}\tilde{B}$ and $\tilde{W}_3\tilde{W}_3$ respectively. μ is the coupling of the $H_1 - H_2$ term the superpotential. Finally $tg\beta$ denotes the ratio of the VEV's of the H_2 and H_1 scalar fields.

In general M_1 and M_2 are two independent parameters, but if one assumes that a grand unification scale takes place, then at the grand unification $M_1 = M_2 = M_3$, where M_3 is the gluino mass at that scale. Then at M_W one obtains:

$$\begin{aligned} M_1 &= \frac{5}{3} tg^2\theta_w M_2 \simeq \frac{M_2}{2}, \\ M_2 &= \frac{g_2^2}{g_3^2} m_{\tilde{g}} \simeq m_{\tilde{g}}/3, \end{aligned} \tag{1}$$

where g_2 and g_3 are the SU(2) and SU(3) gauge coupling constants, respectively.

The relation (1) between M_1 and M_2 reduces to three the number of independent parameters which determine the lightest neutralino composition and mass: $tg\beta, \mu$ and M_2. Hence, for fixed values of $tg\beta$ one can study the neutralino spectrum in the (μ, M_2) plane. The major experimental inputs to exclude regions in this plane are the request that the lightest chargino be heavier than $M_Z/2$ (actually, this limit has been now pushed up to ~ 80 GeV with the advent of LEP200) and the limits on the invisible width of the Z hence limiting the possible decays $Z \rightarrow \chi\chi, \chi\chi'$.

Moreover if the GUT assumption is made, then the relation (1) between M_2 and $m_{\tilde{g}}$ implies a severe bound on M_2 from the experimental lower bound on $m_{\tilde{g}}$ from Tevatron (roughly $m_{\tilde{g}} > 150GeV$, hence implying $M_2 > 50GeV$). The theoretical demand that the electroweak symmetry be broken radiatively , i.e. due to the renormalization effects on the Higgs masses when going from the superlarge scale of supergravity breaking down to M_W, further constrains the available (μ, M_2) region.

The first important outcome of this analysis is that the lightest neutralino mass exhibits a lower bound of roughly 20 GeV . The prospects for an improvement of this lower limit at LEP 200 crucially depends on the composition of χ. If χ is mainly a gaugino, then it is difficult to go beyond 40 GeV for such a lower bound, while with a χ mainly higgsino the lower bound can jump up to $m_\chi > M_W$ at LEP 200.

Let us focus now on the role played by χ as a source of CDM. χ is kept in thermal equilibrium through its electroweak interactions not only for $T > m_\chi$, but even when T is below m_χ. However for $T < m_\chi$ the number of $\chi's$ rapidly decreases because of the appearance of the typical Boltzmann suppression factor exp $(-m_\chi/T)$. When T is roughly $m_\chi/20$ the number of χ diminished so much that they do not interact any longer, i.e. they decouple. Hence the contribution to Ω_{CDM} of χ is determined by two parameters: m_χ and the temperature at which χ decouples (T_D). T_D fixes the number of $\chi's$ which survive. As for the determination of T_D itself, one has to compute the χ annihilation rate and compare it with the cosmic expansion rate [8].

Several annihilation channels are possible with the exchange of different SUSY or ordinary particles, $\tilde{f}$, H, Z, etc. . Obviously the relative importance of the channels depends on the composition of χ. For instance, if χ is a pure gaugino, then the $\tilde{f}$ exchange represents the dominant annihilation mode.

Quantitatively [9], it turns out that if χ results from a large mixing of the gaugino ($\tilde{W}_3$ and $\tilde{B}$) and higgsino ($\tilde{H}_1^0$ and $\tilde{H}_2^0$) components, then the annihilation is too efficient to allow the surviving χ to provide Ω_0 large enough. Typically in this case $\Omega_0 < 10^{-2}$ and hence χ is not a good CDM candidate. On the contrary, if χ is either almost a pure higgsino or a pure gaugino then it can give a conspicuous contribution to Ω_0

In the case χ is mainly a gaugino (say at least at the 90% level) what is decisive to establish the annihilation rate is the mass of $\tilde{f}$. LEP 200 will be able, hopefully, to test slepton masses up to M_W. If sfermions are light, the χ annihilation rate is fast and the Ω_χ is negligible. On the other hand, if $\tilde{f}$ (and hence $\tilde{l}$, in particular) is heavier than 150 GeV, the annihilation rate of χ is sufficiently suppressed so that Ω_χ can be in the right ball-park for Ω_{CDM}. In fact if all the $\tilde{f}'s$ are heavy, say above 500 GeV and for $m_\chi \ll m_{\tilde{f}}$, then the suppression of the annihilation rate can become even too efficient yielding Ω_χ unacceptably large.

Let us briefly discuss the case of χ being mainly a higgsino. If the lightest neutralino is to be predominantly a combination of $\tilde{H}_1^0$ and $\tilde{H}_2^0$ it means that M_1 and M_2 have to be much larger than μ. Invoking the relation (1) one concludes that in this case we expect heavy gluinos, typically in the TeV range. As for the number of surviving $\chi's$ in this case, what is crucial is whether m_χ is larger or smaller than M_W. Indeed, for $m_\chi > M_W$ the annihilation channels $\chi\chi \to WW, ZZ, t\bar{t}$ reduce Ω_χ too much. If $m_\chi < M_W$ then acceptable contributions of χ to Ω_{CDM} are obtainable in rather wide areas of the $(\mu - M_2)$ parameter space. Once again we emphasize that the case χ being a pure higgsino is of particular relevance for LEP 200 given that in this case χ masses up to M_W can be explored.

In the minimal SUSY standard model there are five new parameters in addition to those already present in the non–SUSY case. Imposing the electroweak radiative breaking further reduces this number to four. Finally, in simple supergravity realizations the soft parameters A and B are related. Hence we end up with only three new independent parameters. One can use the constraint that the relic χ abundance provides a correct Ω_{CDM} to restrict the allowed area in this 3–dimensional space. Or, at least, one can eliminate points of this space which would lead to $\Omega_\chi > 1$, hence overclosing the Universe. For χ masses up to 150 GeV it is possible to find sizable regions in the SUSY parameter space where Ω_χ acquires interesting values for the DM problem. A detailed discussion on this point is beyond the scope of this talk. The interested reader can find a thorough analysis in the review of Ref.[7] and the original papers therein quoted.

3 Light Gravitinos as Dark Matter

An alternative scenario to that we discussed in the previous Section is based on the possibility that SUSY is broken in a "secluded" sector at a much lower scale with gauge instead of gravitational forces responsible for conveying the breaking of SUSY to the observable sector. This scenario had already been critically considered in the old days of the early constructions of SUSY models and has raised a renewed interest recently with the proposal by Refs.[10,11,12], where some guidelines for the realization of low-energy SUSY breaking are provided. In these schemes, the gravitino mass ($m_{3/2}$) loses its role of fixing the typical size of soft breaking terms and we expect it to be much smaller than what we have in models with a hidden sector. Indeed, given the well-known relation [5] between $m_{3/2}$ and the scale of SUSY breaking $\sqrt{F}$, i.e. $m_{3/2} = O(F/M)$, where M is the reduced Planck scale, we expect $m_{3/2}$ in the keV range for a scale $\sqrt{F}$ of $O(10^6$ GeV) that has been proposed in models with low-energy SUSY breaking in a visible sector.

Models with light gravitinos have recently attracted much phenomenological attention [13] also in relation with the "famous" anomalous CDF $\gamma\gamma e^+e^-$ event. Needless to say, one should be very cautious in drawing any implication from just a single event. However, with this reservation in mind, we can surely state that this event finds an attractive interpretation in the decay of neutralinos having light gravitinos in the final products.

In the following we briefly report some implications of SUSY models with a light gravitino (in the keV range) in relation with the dark matter (DM) problem. We anticipate that a gravitino of that mass behaves as a warm dark matter (WDM) particle, that is, a particle whose free streaming scale involves a mass comparable to that of a galaxy, $\sim 10^{11-12} M_\odot$.

Suppose that the gravitinos were once in thermal equilibrium and were frozen out at the temperature T_f during the cosmic expansion. It can be shown that the density parameter Ω_0 contributed by relic thermal gravitinos is

$$\Omega_0 h^2 = 1.17 \left(\frac{m_{3/2}}{1\mathrm{keV}}\right) \left(\frac{g_*(T_f)}{100}\right)^{-1}, \tag{2}$$

where $g_*(T_f)$ represents the effective massless degrees of freedom at the temperature T_f. Therefore, a gravitino in the above-mentioned keV range provides a significant portion of the mass density of the present Universe.

As for the redshift at which gravitinos becomes non relativistic, it corresponds to the epoch at which their temperature becomes $m_{3/2}/3$. That is,

$$\begin{aligned} Z_{nr} &\simeq \left(\frac{g_*(T_f)}{g_{*S}(T_0)}\right)^{1/3} \frac{m_{3/2}/3}{T_0} \\ &= 4.14 \times 10^6 \times \left(\frac{g_*(T_f)}{100}\right)^{1/3} \left(\frac{m_{3/2}}{1\mathrm{keV}}\right), \end{aligned} \tag{3}$$

where $T_0 = 2.726\,K$ is the temperature of the CMB at the present time. Once Z_{nr} is known, one can estimate the free streaming length until the epoch of the matter-radiation equality, λ_{FS}, which represents a quantity of crucial relevance for the formation of large–scale cosmic structures. If $v(t)$ is the typical velocity of a DM particle at the time t, then

$$\begin{aligned}\lambda_{FS} &\equiv \int_0^{t_{eq}} \frac{v(t)}{a(t)} dt \\ &= 2t_0 \times \frac{Z_{eq}^{1/2}}{Z_{nr}} [1+\ln(Z_{nr}/Z_{eq})] \\ &= 6.08\times 10^5 \times Z_{nr}^{-1} \mathrm{Mpc}[1+\ln(Z_{nr}/2.32h^2\times 10^4)]\,,\end{aligned} \tag{4}$$

where Z_{eq} is the redshift of matter–radiation equality. Accordingly, the free-streaming length for the thermal gravitinos is about 1Mpc (for $Z_{nr}\sim 4\times 10^6$), which in turn corresponds to $\sim 10^{12}M_{\odot}$, if it is required to provide a density parameter close to unity. This explicitly shows that light gravitinos are actually WDM candidates. We also note that, taking $h=0.5$, the requirement of not overclosing the Universe turns into $m_{3/2}\lesssim 200\,eV$. Quite interestingly, this constraint on $m_{3/2}$ is of the same order as the upper limit of $\sim 250\,eV$, based on the light gravitino interpretation of the mentioned CDF event[13]. Moreover, if we rely on the lower bound, $m_{\tilde{g}}\geq 30$ eV, based on observations about the energy loss from the supernova SN1987A[14], then we conclude that $\Omega_0\gtrsim 0.15$ should be provided by light gravitinos.

However, critical density models with pure WDM are known to suffer for serious troubles[15]. Indeed, a WDM scenario behaves much like CDM on scales above λ_{FS}. Therefore, we expect in the light gravitino scenario that the level of cosmological density fluctuations on the scale of galaxy clusters ($\sim 10\,h^{-1}$Mpc) to be almost the same as in CDM. As a consequence, the resulting number density of galaxy clusters is predicted to be much larger than what observed[16]).

In order to overcome these problems one should follow in principle the same patterns as for improving the standard CDM scenario: *(a)* tilting the primordial spectrum to $P(k)\propto k^n$ with $n<1$, also assuming a large ($\sim$ 20%) baryon fraction [17]; *(b)* decreasing the density parameter to $\Omega_0<1$, with the possible introduction of a cosmological constant term to make the spatial curvature negligible, as predicted by standard inflationary models; *(c)* add a second DM component, made by particles with a much larger λ_{FS},so as to suppress fluctuations on the cluster scales.

As for the scenario *(b)*, differently from what happens in the CDM case, the choice of Ω_0 reflects also in a significant variation of λ_{FS}. Therefore, the value of Ω_0 does not only affect the power spectrum of fluctuations through the determination of the equivalence epoch, but also changes the small–scale fluctuation power due to the change in λ_{FS}.

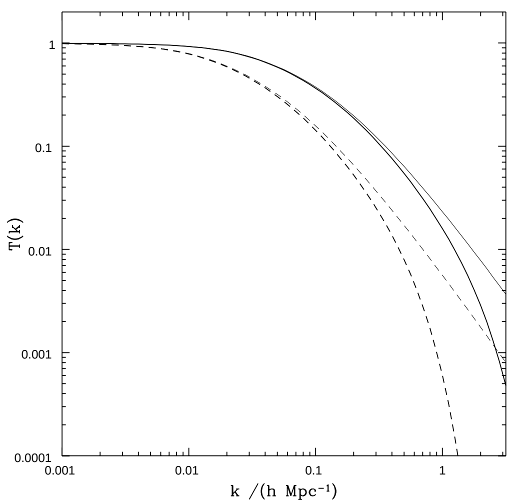

Figure 1: Transfer functions for WDM (heavier lines) and CDM (lighter lines) by using the fitting function by Bardeen et al. Solid and dashed curves are for $(\Omega_0, h) = (1, 0.5)$ and $(0.3, 0.7)$ respectively. The free–streaming scale for WDM is chosen to be $\lambda_{FS} = 0.5\,\Omega_0 h^{-1}$Mpc.

We plot in Figure 1 the transfer functions $T(k)$ for density perturbations at the outset of recombination for WDM and CDM models, in the case of $\Omega_0 = 1$ and $\Omega_0 = 0.3$ (see caption). This plot is based on the fitting expression for $T(k)$ provided by Bardeen et al. [18]. We take $\lambda_{FS} = 0.5\Omega_0^{-1} h^{-2}$ Mpc, which is appropriate for light gravitinos (cf. eq.5). As expected, cold and warm scenarios coincide on scales above λ_{FS}, while on smaller scales the fluctuation power is exponentially suppressed for WDM by free–streaming.

As for the scenario *(c)*, one can envisage for instance the possibility that in addition to the warm light gravitinos there are massive light neutrinos (in the eV range) considering a scheme with a mixed WDM+ HDM[19]. Another possibility, that we have recently investigated together with M. Yamaguchi [20], takes into account the fact that we have a "secondary population" of gravitinos, which result from the decay of the next-to-the-lightest superparticle (NSP), presumably the lightest neutralino. They have a non–thermal phase–space distribution and exhibit features for the structure formation which are similar to those of a standard hot light neutrino in the tens of eV range. From our analysis it turned out that viable MDM realizations within the frame of light gravitinos lead to characteristic features both in the cosmological and particle physics contexts, making these models testable against astrophysical observations and future accelerator experiments. In particular, on the astrophysical side, we find a relatively large ^{4}He abundance (corresponding to slightly more than three neutrino species at nucleosynthesis), a suppression of high redshift galaxy formation with respect to the cold dark matter (CDM) scenario and a free-streaming scale of the non-thermal ("secondary") gravitinos independent of $m_{3/2}$, but sensitive to the NSP mass (with important consequences on the large scale structure formation). As for the particle physics implications, the implementation of a MDM scheme imposes severe constraints on the SUSY particle spectrum. For instance the lightest neutralino (NSP) should be an essentially pure gaugino and sfermions have to be rather heavy (in the TeV range).

As a general comment, it is worth stressing once more that, from the point of view of the formation of cosmic structures, all the scenarios based on a dominant WDM component differ from their CDM counterpart only on scales below λ_{FS} of gravitinos. Therefore, the suppression of fluctuation power on galactic scales leads in general to a later galaxy formation. This is potentially a critical test for any WDM–dominated scheme, the abundance of high–redshift galaxies having been already recognized as a non trivial constraints for several DM models. It is however clear that quantitative conclusions on this point would at least require the explicit computation of the fluctuation power spectrum for the whole class of WDM scenarios, a project on which we are currently working in collaboration with E. Pierpaoli.

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